Information Disclosure in Markets:*
Auctions, Contests and Matching Markets

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Abstract

We study the impact of information disclosure on equilibrium properties in a model of a two-sided matching market that incorporates a large class of market design environments. In this model, each agent first privately observes an informative, but potentially noisy, signal about his private type. The agents then enter a matching stage in which they choose signaling investments to compete for match partners. In order to study the impact of information disclosure, we introduce a novel criterion that orders signals in terms of their informativeness. We show that information disclosure increases the expected total match output, but may also increase wasteful signaling investments due to amplified competition within groups. The second effect may dominate, leading to a decrease in expected welfare. Disclosure effects on equilibrium properties depend on whether information is disclosed to agents on the short or on the long side of the market. Applications to auctions, contests, and matching markets are discussed.

Keywords: Matching, Information disclosure, Informativeness, Stochastic orderings, Auctions.

JEL classification: C78, D44, D82, D83.

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1 Introduction

In most market environments the information available to market participants significantly influences agents' behavior and the market outcome. Examples include auctions, contests, and various matching markets, among them school choice, college admission and labor markets. In auctions, bidding behavior as well as the revenue of an auctioneer depend on the information available to bidders about the object being sold. Feedback provided by a company to their workers affects workers' effort in promotion tournaments, and the precision of grading systems in high-schools influences the outcome of college admission.

In the recent auction literature, the effects of the precision level of information available to bidders on bidding behavior, efficiency, and expected revenue in an auction have been studied extensively. The resulting implications for auction design have been discussed.[1] It is therefore surprising that this important topic has been little studied and is still not well understood in other market design environments.

In this paper we address this problem and analyze the effects of information disclosure on agents' behavior, the resulting assignment, and welfare implications in a two-sided matching market model. We explain how our results map to various of the aforementioned market design environments, and discuss the implications for auctions, contests and matching markets.

We consider a model of a two-sided, one-to-one matching market with a finite number of agents on each side of the market. We refer to the two groups of agents as *workers* and *firms*. These terms are only used to distinguish the two groups of agents. They can represent for example workers and firms, students and schools, competitors and prizes, or bidders and objects in auctions.

Our model is a modification of the marriage market formulated in Becker (1973), with two-sided incomplete information. Firms have private information about their types, whereas workers are a priori uncertain about their own types. The model has two stages: an *information stage* followed by a *matching stage*. In the information stage, workers obtain an informative, but typically noisy, private signal about their individual type and update their beliefs accordingly.

The signal realizations in the information stage determine the private information of workers. Information disclosure means that workers obtain more informative signals, which results in a higher information level of workers. In order to study the effects of information disclosure on equilibrium properties in the second-stage matching game, we introduce a

[1] Examples include Persico (2000) and Shi (2012) who study information acquisition, whereas the focus in Bergemann and Pesendorfer (2007), Esö and Szentes (2007), Ganuza and Penalva (2010) and Ganuza and Penalva (2014) is on information disclosure.

criterion, which we call *single-crossing precision*, that orders signals in terms of their informativeness. This precision criterion is similar to those introduced in Ganuza and Penalva (2010). It also uses the insight that a more informative signal yields a more dispersed distribution of posterior estimates.

In the matching stage, agents take part in a matching game, in which they choose investments to compete for match partners. Investments are non-productive and serve as observable signals about the private types of agents. This matching game was introduced and studied by Hoppe et al. (2009) who refer to it as a *matching tournament*.[2] They prove the existence of a separating equilibrium in which agents are matched positively assortatively according to their investments. In our analysis, we focus on this equilibrium.

We find that increasing the level of information available to workers increases the expected total match output as well as the expected investments of firms, whereas the expected investments of workers may decrease. Workers always profit from information disclosed to them, whereas this disclosure may negatively affect the welfare of firms. We show that the second effect may be so strong that expected aggregate welfare is decreased, but also identify conditions that guarantee that the expected aggregate welfare is increased by a higher information level available to workers. For the case in which disclosing more precise information is costly, we characterize the worker-optimal and the socially optimal levels of information. The socially optimal level of information maximizes expected aggregate welfare, whereas the worker-optimal level only takes into account the utilities of workers and maximizes the welfare of this group.

The results are driven by two, possibly opposing, effects of information. On the one hand, a higher information level of market participants allows for a better assignment in the matching game, which increases expected total match output. On the other hand, disclosing information to agents also amplifies competition within groups, which may result in increased (wasteful) investments in the matching game. Our results indicate that the second effect may dominate, resulting in decreased welfare.

These two effects are based on the following feature of two-sided markets in which agents have private information: Agents impose externalities not only on agents within their group, but also on agents on the other side of the market. In particular, a worker imposes a positive externality on firms by providing a match opportunity. However, he also imposes a negative externality on them, since more or better match opportunities lead to increased competition among firms, which results in higher expected investments.

[2] Such a tournament is a generalization of a contest in which prizes are replaced by matching opportunities. A similar model with a continuum of agents is discussed in Hopkins (2012).

We apply the results that we establish in our general framework, to discuss the implications of information disclosure in various market design settings, focusing on auctions, contests, and matching markets. Depending on the application, agents' investments are interpreted as wasteful signaling costs or as (monetary or non-monetary) transfers to a third party. In each of the applications we highlight certain features of our results.

The most straightforward application of our model are two-sided matching markets, and the implications of our results yield new insights for these settings. Even though there seems to be a broad agreement that the level of information of agents in two-sided matching markets is an important factor which influences agents' behavior and the market outcome, these informational effects are poorly understood theoretically.[3] Interpreting our results as they apply to two-sided matching markets yields one of our main contributions: To our knowledge, we provide the first study of the impact of information disclosure, and the level of information available to market participants, on the outcome in a two-sided matching market. The main observations are that disclosing information may decrease expected aggregate welfare, and that the effects depend on whether information is disclosed to the agents on the short or on the long side of the market.

Our results can also be applied to illustrate the impact of information disclosure in contests or rank-order tournaments, for example through feedback systems in organizations. The flexibility of our framework provides two distinct ways to project our general model to contests, each yielding different insights and predictions. This feature allows us to obtain some of the existing results in the contest literature as special cases of our results. More importantly, our analysis provides new insights for the role of feedback systems in contest. We show that the effects of information disclosure in contests depend on the ratio of competitors to prizes, and on the prize-structure. For example, in promotion tournaments with a large pool of workers and only a few available positions, providing information to workers increases overall effort, whereas this is not necessarily the case if the ratio of workers and promotions is relatively balanced. Our results moreover suggest that information management through feedback systems may serve as an powerful element of contest design. We derive predictions about which feedback systems are to be expected in different organizational structures.

Our methodological contribution is highlighted in the application of our results to auc-

[3] A reason for this may be that most of the theoretical analysis of matching markets studies complete information models, in which agents know their preferences over potential match alternatives. The two aspects, that agents have private information about their characteristics and may moreover be uncertain about their own characteristics, are hardly captured by the theoretical models in the literature. Our model incorporates both of these aspects and therefore takes a first step towards a theoretical analysis of the effects of private information and the information level of market participants on the equilibrium outcome in matching markets.

tions. We discuss how the statistical methods that we use in this paper provide an alternative way to prove the results of Ganuza and Penalva (2010) on information disclosure in auctions. To further illustrate the potential of these statistical methods, we show how they can be used to strengthen and generalize the results of Ganuza and Penalva (2010).

Outline The rest of the paper is organized as follows. In Section 2 we present the model. In Section 3 we characterize equilibrium properties for an exogenously given level of information. In Section 4, we introduce single-crossing precision, a novel criterion to measure the informational content of signals. The effects of information disclosure and the resulting higher information level of market participants are discussed in Section 5. Section 6 contains our results on the worker-optimal and socially optimal levels of information. The implications of our results for auctions, contests, and matching markets are presented in Section 7. Related literature is discussed in Section 8, and Section 9 concludes. All proofs are relegated to the appendix. In Appendix C, we also briefly discuss different precision criteria.

2 The Model

Consider a two-sided matching market with a finite number of agents. We refer to the two groups of agents, constituting the two sides of the market, as *workers*, $\mathcal{I} = \{1, \dots, n\}$, and *firms*, $\mathcal{J} = \{1, \dots, k\}$. These terms are only used to distinguish the two groups. Depending on the application they may represent for example, workers and firms, men and women, students and colleges, or competitors and prizes.

The types of workers, x_i, and firms, y_i, are determined by iid draws from the interval $[0, \overline{x}]$, respectively $[0, \overline{y}]$.[4] Agents' types are independently distributed with prior distribution F_X for workers and F_Y for firms. We assume throughout the paper that $F_X(0) = F_Y(0) = 0$ and F_X and F_Y are continuously differentiable with positive densities, $f_X > 0$ and $f_Y > 0$, on the support.

There is incomplete information on both sides of the market: Firms' types y_j, are private information to the firms, whereas workers' do not know their types ex-ante. The distributions F_X and F_Y are common knowledge.[5]

If worker i is matched with firm j, each agent obtains match payoff $x_i y_j$, unmatched agents produce zero output. In other words, the match value function is $v(x, y) = 2xy$ and match output is split equally among match partners.

[4] If $\overline{x}$ or $\overline{y}$ equal infinity, types are drawn from $[0, \infty)$.

[5] We consider this model in order to simplify notation. It is straightforward to extend the analysis to the case in which agents on both sides of the market are uncertain about their types.

In this model, under complete information, all workers agree on the ranking of firms and vice versa. Our match value function is supermodular, which implies that *positive assortative matching* is the allocation that maximizes expected match output and, moreover, is the only stable matching.

We consider a two-period model which consists of an *information stage* followed by a *matching stage*. In the first period, workers obtain an informative, private signal about their individual types and update their beliefs accordingly. Agents then enter the matching stage, in which they compete for match partners.

2.1 Information Stage

In the first period, the *information stage*, every worker observes a private signal realization from an information technology S.

An information technology is a signal S, with typical realizations $s \in [0, \overline{s}]$, which is characterized by a family of conditional distributions $\{G(\cdot|x)\}_{x \in X}$ of signal realization.

$$G(s|x) := Pr(S \leq s|X = x)$$

is the probability that a worker with type x receives a signal realization $s' \leq s$. We assume that for every $x \in X$, $G(\cdot|x)$ is absolutely continuous, that is, admits a density function $g(\cdot|x)$ almost everywhere. Together with the prior distribution F_X, an information technology induces a joint distribution on (X, S), a so-called *information structure*. Agents update their beliefs according to Bayes' rule. With a slight abuse of notation, the posterior distribution of X conditional on $S = s$ is $G(\cdot|s)$, and the resulting conditional expectation is

$$\widehat{X}(s) = E\left[X|s\right] = \int_{\mathcal{X}} x \mathrm{d}\, G(x|s).$$

We denote the marginal distribution of S by G.

We assume that high signals are *more favorable* than low signals in the sense of Milgrom (1981). This condition implies that workers with high types are more likely to observe a high signal realization than workers with low types. A high signal thus indicates a higher underlying type of the agent than a low signal.

Assumption 1 (Monotone Signals)**.** *For all signal realizations $s, s' \in S$ with $s' > s$, signal realization s' is* more favorable *than s. That is, for every non-degenerate prior distribution F on X, if $s' > s$, then the posterior distribution $G(\cdot|s')$ dominates $G(\cdot|s)$ in terms of first-order stochastic dominance, $G(\cdot|s') \geq_{FOSD} G(\cdot|s)$.*

This assumption implies that posterior estimates are strictly increasing in signal realizations:

$$E[X|s'] > E[X|s], \text{ for every } s' > s.$$

If signal S is characterized by conditional densities $\{g(\cdot|x)\}_{x\in X}$, then Assumption 1 is equivalent to the strict monotone likelihood ratio property.[6]

For a given prior distribution F, every information technology S results in a distribution of posterior estimates, represented by a random variable $\widehat{X} := E[X|S]$. Given Assumption 1, the function $\widehat{X} : S \to \mathbb{R}^+$, with $\widehat{X}(s) = E[X|s]$ is strictly increasing in s, which implies that there exists an inverse function, $\widehat{X}^{-1}$. The distribution function of the posterior estimates is

$$H(\hat{x}) = G\left(\widehat{X}^{-1}(\hat{x})\right) = \int_{\mathcal{X}} G\left(\widehat{X}^{-1}(\hat{x})|x\right) \mathrm{d}\, F(x),$$

with quantile function $H^{-1}(u) = \inf\{\hat{x}|H(\hat{x}) \geq u\}$ for $u \in [0,1]$.

We provide two examples of information technologies that are commonly used in the literature.

Example 1 (Truth-or-noise technology)**.**
Suppose that X is the set of states with prior distribution F_X. A truth-or-noise technology provides with some probability $\alpha \in [0,1]$ a perfectly informative signal $s = x$ and with probability $(1-\alpha)$ pure noise, independently drawn from prior distribution F_X. The receiver cannot distinguish which kind of signal he observes. For signal realization s, the conditional expected value is $E[X|s] = \alpha s + (1-\alpha)E[X]$. △

Example 2 (Normal Experiments)**.**
Suppose that workers' types are normally distributed, $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$, and that signal S is given by $S = X + \epsilon$, with a normally distributed noise term, $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. It follows that signals are normally distributed, $S \sim \mathcal{N}(\mu_X, \sigma_X^2 + \sigma_\epsilon^2)$. The posterior estimates are given by

$$\widehat{X}(s) = \frac{\sigma_\epsilon^2}{\sigma_X^2 + \sigma_\epsilon^2}\mu + \frac{\sigma_X^2}{\sigma_X^2 + \sigma_\epsilon^2}s,$$

thus linear in S, and again normally distributed. △

[6] The collection $\{g(\cdot|x)\}_{x\in X}$ has the strict *monotone likelihood ratio property* (MLRP) if for every $x > x'$, $\frac{g(s|x)}{g(s|x')}$ is strictly increasing in s.

In our setting, workers do not know their types a priori. Thus, in the matching stage, workers can only condition their decisions on the information obtained in the information stage. For a given prior distribution, F_X, the information technology S determines the distribution of conditional expected types of workers in the matching stage, $H^S(\hat{x})$, which is common knowledge. The individual expected types conditional on the private signal realizations are $\hat{x}_i = E[X|s_i]$. With a slight abuse of terminology, we refer to them as workers' *posterior types*. They are private information of the workers. Similarly, for firms, type y_j is private information to firm j. Given the linearity of our model, if a worker with posterior type $\hat{x}_i$ is matched to a firm with type y_j, the expected match output for each of the match partners is $\hat{x}_i y_j$.

2.2 Matching Stage

In the second period, the *matching stage*, all agents simultaneously choose an individual investment that serves as a costly signal of their type. Agents on each side of the market are ranked based on their investments to then be matched positively assortatively. In case of equal investments, we assume random tie-breaking. Under this assignment rule the worker with the highest investment will be matched to the firm with the highest investment, the agents with the second highest investments in each of the groups will be matched, and so on.[7] If worker i invests b and is matched with firm j, his payoff is

$$u_i\left((x_i, b), y_j\right) := x_i y_j - b.$$

A (pure) strategy for a firm is a measurable function from the set of types Y to non-negative investments $\mathbb{R}_0^+$. For workers it is a mapping from signal realizations S to investments. The solution concept is Bayesian Nash equilibrium.

Remark. The discussion at the end of Subsection 2.1 illustrates that after the information stage the situation is as if agents on both sides of the market have private information about their types. We explicitly model the information stage because we are interested in the comparative statics effects that correspond to changes in the informativeness of the information technology of workers. One contribution of the paper is to identify the effect of a more precise information technology on the distribution of posterior types of workers (Section 4), to then study the comparative static effects that result from these changes

[7] A matching mechanism which would yield this outcome is, for example, the worker-, or firm-proposing deferred acceptance algorithm, assuming that agents rank their potential match-partners according to the observed investments. The assignment rule is also the natural extension to auctions, in which the highest bidder obtains the object.

(Section 5), and discuss the implications for various applications (Section 7).

2.3 Market Design Settings Captured by the Model

We now briefly discuss how the model captures various important market design settings. Table 1 provides a summary.

Matching Markets The model represents a two-sided, one-to-one matching market, in which agents on each side have homogeneous preferences about match partners. There is two-sided incomplete information and agents invest in non-productive signaling à la Spence (1973) to compete for match partners.[8] Investments are wasteful. They solely serve as an observable signal of the agent's unobservable type but do not have an effect on the match output. Agents of each group are ranked according to their observable signaling investments to then be matched positively assortatively.

Auctions The standard private values auction setting can be mapped into a special case of our model. There is only one firm, with type $Y \equiv 1$, interpreted as the auction platform which sells an object of commonly known quality 1. The workers represent the bidders in the auction, with their types corresponding to their valuations of the object, and investments corresponding to their bids. The seller is represented by a third party, which collects the bids. This interpretation yields an all-pay auction. By the revenue equivalence theorem, the results which we present in Section 5 and Section 6 apply to all standard auctions that implement the efficient allocation.

Contests The model also captures a rank-order tournament or contest setting in the following sense: Consider the firms as passive agents, who are the prizes in a contest, with commonly known values $\eta_{1:k} \geq \cdots \geq \eta_{k:k}$. Workers represent the competitors who participate in the contest, workers' types correspond to their abilities, and investments capture the exerted effort.

3 Equilibrium Characterization

In this section we characterize the equilibrium in the second-period matching game on which we will focus in our analysis. It is easy to see that there exist multiple equilibria, among them

[8] In contrast to Spence (1973), here agents do not have different cost-types but high-type agents receive a higher payoff from a particular match than low-type agents.

[9] The passive firm could for example represent an auction platform. The seller is a third party who collects the investments.

	workers	firms	types	investments
auctions	bidders	*–passive–*[9]	valuations	bids
contests/ tournaments	workers, competitors	promotions, prizes	productivity, abilities	effort
matching markets	students, workers	schools, jobs	characteristics	signaling investments

Table 1: Examples of environments captured by the model.

a pooling equilibrium in which all agents choose zero investments and the assignment is random. We say that an equilibrium is *symmetric* if all workers adopt the same strategy and so do all firms. Strategies are *monotone* if they are given by continuously differentiable, strictly increasing functions. Under monotone strategies, firms' with higher types choose higher investments and workers' investments are increasing in signal realizations. The existence of a symmetric separating equilibrium in monotone strategies follows directly by adapting the results of Hoppe et al. (2009) to our setting. Given Assumption 1 there exists a unique equilibrium of this type.

Theorem 1 (Hoppe et al. 2009)**.** *Given the assumptions in Section 2, in the second-period matching game, there exists a unique symmetric separating equilibrium in monotone strategies.*

In our analysis, we will focus on this separating equilibrium. In this equilibrium, the positive assortative matching with respect to the (posterior) types of agents is implemented, all agents of a group adopt the same strategies, and high-type agents choose higher investments than low-type agents.

Remark. There are various reasons why it is natural to focus on the equilibrium of Theorem 1. This separating equilibrium is the unique equilibrium that is monotone in signal realizations. Moreover, it implements the unique stable (and core) matching, given the information available in the market after the information stage. It is also the natural extension to the efficient allocation in auctions and contests.

Before we can provide some intuition for the equilibrium and the formulas for expected total output, investments and welfare in equilibrium, we need to introduce some more notation.

For a sample $X_1, \ldots, X_n$ let

$$X_{1:n} \geq_{FOSD} \cdots \geq_{FOSD} X_{n:n}$$

be the corresponding order statistics, where $\geq_{FOSD}$ indicates first-order stochastic dominance. The random variable $X_{i:n}$ represents the distribution of the i^{th} highest among n iid draws. In particular, $X_{1:n} = \max\{X_1, \ldots, X_n\}$.[10]

For given market sizes n, k, priors F_X, F_Y and information technology S set

$$\mu^S_{i:n} := E\left[\widehat{X}_{i:n}\right] \quad \text{and} \quad \eta_{i:k} := E\left[Y_{i:k}\right].$$

That is, $\mu^S_{i:n}$ denotes the expected value of the i^{th} order statistics of the posterior types, given information technology S.

To obtain some intuition for this equilibrium it is instructive to analyze the reduced game faced by agents on either side of the market separately, and relate equilibrium investments to Vickrey-payments.[11] This interpretation also more precisely demonstrates how the model is a natural extension of a standard auction setting.

Suppose firms adopt separating strategies, and consider the situation for workers after the information stage. In this case, the problem faced by workers in the second-period matching game is as if they are in a contest competing for k heterogeneous prizes, where the values of the prizes are determined by an agent's posterior type and the expected values of the highest, second-highest, third-highest,... types of firms. To be precise, for a worker with posterior type $\hat{x}$, the values of the prizes are $\hat{x} \cdot \eta_{1:k}, \ldots, \hat{x} \cdot \eta_{k:k}$. The assignment in the matching stage is positive assortative with respect to investments. In the corresponding contest faced by workers, this allocation rule thus prescribes that prizes be allocated to the agents in order of their investments, where the agent with the highest investment receives the highest price. Our model is linear, and it is well-known that in such an environment expected payoffs of agents are fully specified by the allocation rule, and the expected payoff of the lowest type.[12] By the revenue equivalence theorem, it follows that expected investments must be the same as in a VCG-mechanism.[13] In the VCG-mechanism, each worker must pay the amount equal to the negative externality he imposes on the other workers. For a profile of signal realizations $s_1, \ldots, s_n$, after appropriate relabeling, let the corresponding posterior types be $\hat{x}_1 \geq \cdots \geq \hat{x}_n$. We refer to the worker receiving the i^{th}-highest signal, as *(posterior) type* i. The presence of type i does not affect workers who receive a higher signal than himself,

[10] Hereby, we adopt the notation which is used in most of the economics literature. It should be noted that, by contrast, the standard convention in statistics is to denote the highest order statistic by $X_{n:n}$.

[11] This was also pointed out by Hoppe et al. (2009). Adapting their results to our model yields the equilibrium properties summarized in Table 2.

[12] A worker who receives signal realization 0 does not invest and is matched to the lowest firm with certainty. His expected payoff is $E\left[X|0\right] \cdot \eta_{n:k}$ (which is 0 if $n > k$).

[13] The well-known Vickrey-Clarke-Groves (VCG) mechanisms due to Vickrey (1961), Clarke (1971) and Groves (1973)

	workers	**firms**
expected total output	$O = 2\sum_{i=1}^{\min} \mu_{i:n}^S \cdot \eta_{i:k}$	
expected total investments	$T_w = \sum_{i=1}^{\min} i\big(\eta_{i:k} - \eta_{i+1:k}\big)\mu_{i+1:n}^S$	$T_f = \sum_{i=1}^{\min} i\big(\mu_{i:n}^S - \mu_{i+1:n}^S\big)\eta_{i+1:k}$
expected welfare of workers/firms	$W_w = \sum_{i=1}^{\min} i \cdot \big(\mu_{i:n}^S - \mu_{i+1:n}^S\big)\eta_{i:k}$	$W_f = \sum_{i=1}^{\min} i \cdot \big(\eta_{i:k} - \eta_{i+1:k}\big)\mu_{i:n}^S$
expected aggregate welfare	$W = \sum_{i=1}^{\min} i \cdot \big(2\mu_{i:n}^S\eta_{i:k} - \mu_{i+1:n}^S\eta_{i:k} - \mu_{i:n}^S\eta_{i+1:k}\big)$	

Table 2: Formulas for expected total output, expected total investments and welfare for workers and firms, respectively, and expected aggregate welfare, for given prior distributions, information technology S, and market sizes, n, k. Here, $\min := \min\{k, n\}$.

but he imposes a negative externality on all workers receiving a lower signal realization. Each of those workers would be assigned to a higher match-partner if type i were not present. It follows that the expected investment, t_i, of the i^{th} type is:

$$t_i = \sum_{j=i}^{\min\{k,n\}} \mu_{j+1:n} \cdot (\eta_{j:k} - \eta_{j+1:k}). \tag{1}$$

Summing up over all i we obtain the formula for expected total investments of workers,

$$T_w = \sum_{i=1}^{\min\{k,n\}} i\big(\eta_{i:k} - \eta_{i+1:k}\big)\mu_{i+1:n}^S.$$

In the separating equilibrium of Theorem 1 the assignment in the matching stage is positive assortative with respect to agents (posterior) types. It is easy to see that the resulting expected total match output is $O = 2\sum_{i=1}^{\min\{k,n\}} \mu_{i:n}^S \cdot \eta_{i:k}$. Expected total welfare of workers is $W_w = \frac{1}{2}O - T_w$ and the formulas for equilibrium expected total investments and welfare of firms are derived in a similar fashion. Table 2 provides a summary of the formulas.

4 Precision of Information Technologies

We now introduce a novel criterion, which we call *single-crossing precision*, to compare signals in terms of their informational content. In Section 5, we apply this concept to discuss the effects of a higher level of information of workers on total match output, total investments, and welfare.

Given our assumption that signals are monotone, the natural informativeness criterion to use is the concept of *effectiveness* introduced by Lehmann (1988) – Persico (2000) calls this concept *accuracy*. The basic idea behind this concept is that, for a more accurate signal the conditional distributions that characterize the signal are more dependent on the state than for a less accurate signal.[14] Mizuno (2006) shows that for a more accurate signal the resulting distribution of posterior estimates is more dispersed.

As Ganuza and Penalva (2010), we use this observation to define a novel precision criterion in terms of properties of the resulting distribution of posterior estimates.[15] It is based on the following insight: for a completely random signal nothing can be inferred from the signal realization and the resulting posterior estimate is always the ex-ante mean. For a more informative signal, the resulting distribution of posterior estimates will be more responsive to the signal realization and thus result in a more variable distribution of posterior estimates.

The information criterion that we use requires that a more precise signal leads to a more dispersed distribution of posterior estimates in terms of a mean-preserving spread. Moreover, for signals ordered in terms of *single-crossing precision*, we require single-crossing of the quantile functions.

Definition 1. For a given prior F_X and signals S_1, S_2, let H_1^{-1} and H_2^{-1} be the quantile functions of $E[X|S_1]$ and $E[X|S_2]$. Say that signal S_2 is more *single-crossing precise* than S_1, denoted $S_2 \succsim_* S_1$, if

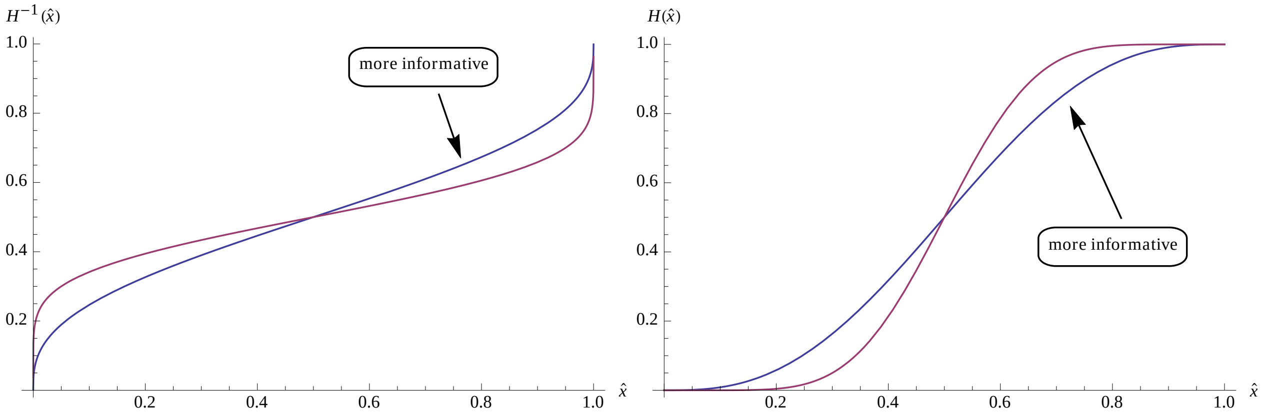
$$\frac{H_2^{-1}(u)}{H_1^{-1}(u)} \text{ is increasing in } u \in (0,1)\,.$$

We say that agents have a *higher information level* if the private signal realizations that agents receive in the information stage originate from a more (single-crossing) precise signal. Single-crossing precision implies that the distribution of posterior estimates resulting from the more precise signal crosses the one resulting from the less precise signal only once and from above. Our criterion is therefore slightly more restrictive than the ordering induced by accuracy.[16] However, many commonly used information structures are ordered in terms of

[14] Signal S_1 is more effective than signal S_2 if $G_{S_1}\left(G_{S_2}^{-1}(s|x)|x\right)$ is increasing in x. Effectiveness applies to monotone decision problems and requires less restrictive conditions than *sufficiency* (Blackwell, 1951) to compare signals in terms of their informativeness.

[15] Our concept is slightly stronger than the concept of *integral precision* in Ganuza and Penalva (2010). Their concept of *supermodular precision* and our concept of *single-crossing precision* are not nested – a formal discussion is provided in Appendix C.

[16] A more accurate signal results in a mean-preserving spread of the distribution of posterior estimates, a property that does not exclude multiple crossings. Signals which are characterized by more or less fine partitions of the state space are typically not ordered in terms of single-crossing precision, since the distributions of posterior estimates may cross multiple times.



(a) *Quantile functions.* (b) *Distribution functions.*

Figure 1: Relation of the quantile and distribution functions of posterior estimates for signals ordered in terms of single-crossing precision.

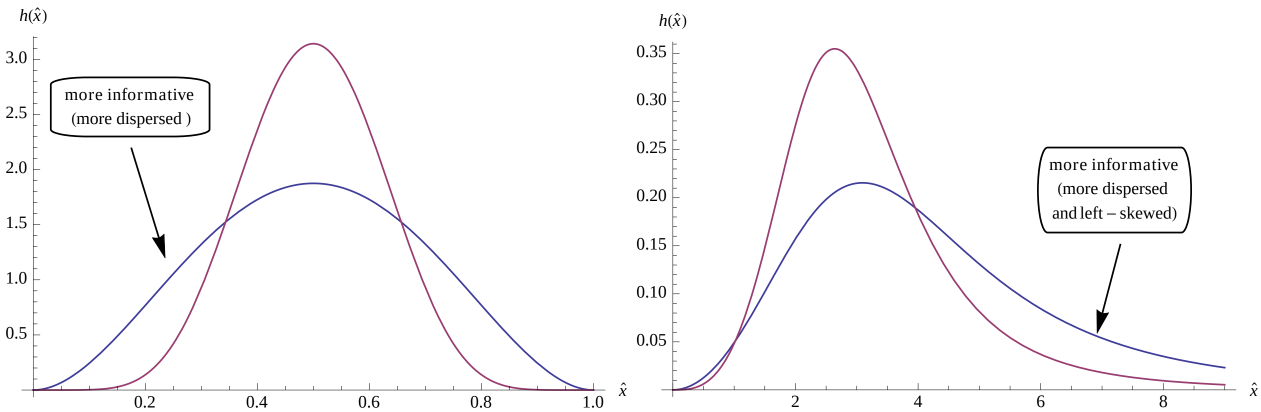
single-crossing precision, among them those in Example 1 and Example 2. For truth-or-noise technologies, signal S_α is more precise than S_β if and only if $\alpha \geq \beta$. For normal experiments a signal with less noise is more precise.

Increasing the precision of a signal in terms of single-crossing precision has two main implications. First, it results in a more dispersed distribution of posterior estimates in terms of a mean-preserving spread. Second, if the distributions of posterior estimates exhibit different levels of skewness, then the more precise signal results in a more left-skewed distribution of posterior estimates.[17] Figure 1 and Figure 2 illustrate properties of the distribution, density and quantile functions of the posterior estimates from signals that are ordered in terms of single-crossing precision.

In order to establish our results, we use the *quantile function representation* for order statistics, which establishes a close link between the properties of the quantile functions and the vector of expected order-statistics.[18] For signals ordered in terms of single-crossing precision the vectors of the expected order statistics of the posterior types satisfy a "single-crossing condition". Switching to a more precise information technology results in an increase of the expected value of the highest order statistics of posterior types, whereas the expected values of lower order statistics will decrease. The following lemma formally states this property.

[17]This feature of our information order is in line with the well-documented observation in the empirical finance literature that many asset return distributions exhibit negative skewness (e.g. Beedles, 1979; Alles and Kling, 1994, and subsequent papers). This property is often attributed to standard practices adopted to release information. Companies tend to release good news immediately (more frequently), whereas bad news are released in clumps. This was first pointed out in Damodaran (1985) and recently discussed in Acharya et al. (2011).

[18]See Arnold et al. (1992).



(a) *Densities – the symmetric case.* (b) *Densities – skewed distributions.*

Figure 2: Density functions of posterior estimates for signals ordered in terms of single-crossing precision. (a) illustrates the case for symmetric functions, (b) the case with different skewness.

Lemma 1. *If information technology S_2 is more single-crossing precise than S_1, then for all $n \in \mathbb{N}$*

$$\frac{\mu_{i:n}^{S_1}}{\mu_{i:n}^{S_2}} \text{ is increasing in } i.$$

5 The Comparative Statics Effects of Higher Precision

How does a change in the information level of workers affect equilibrium behavior of agents and the resulting match outcome? In this section, we address this question and characterize the effects on expected total output, investments and welfare.

In our model workers are a priori uncertain about their own types, and the outcome depends on the private signals that they receive in the information stage. Thus, from the ex-ante perspective, the matching mechanism yields a lottery over all possible matchings of workers to firms. In the two extreme cases, workers either receive no information about their own types, which results in random matching, or they observe a perfectly informative signal and are matched positive assortatively in equilibrium. Since signals are monotone (Assumption 1), workers with high types are more likely to receive a high signal in the information stage than low-type workers. Consequently, if the information level of workers increases, then the probability that workers with high types are matched with firms of a similar ranking increases. This results in a higher expected match output for pairs of high-ranking agents, but may result in a decrease of expected match output for lower-ranked pairs.[19]

[19]This result holds if $E[X|S_2]$ is a mean-preserving spread of $E[X|S_1]$. That is, is suffices to require that signals are ordered in terms of *integral precision* (Ganuza and Penalva, 2010).

Proposition 1. *Let signal S_2 be more precise than signal S_1. Then, expected total match output is increasing in signal precision.*

The result establishes that, at the aggregate level, a higher information level of workers always results in an increase in total match output. This is intuitive. A higher information level of workers allows on average for a better matching.

The effect on expected investments and welfare of workers is harder to characterize. If workers obtain more precise information, the resulting distribution of posterior types is more dispersed. The expected value of the low posterior-type workers decreases. Consequently, for these workers the (marginal) benefit in match output from being matched with a firm of higher ranking reduces and thus also the expected externalities imposed on them by other workers.[20] For high-type workers the effect is reversed and externalities imposed on them are increasing.

At the aggregate level, it is not clear a priori, which of these effects dominates, and some of the following results will depend on the sizes of the two sides of the market, or distributional properties of firms' types.[21]

Theorem 2 (Workers' expected total investments and welfare)**.** *Let signals be ordered in terms of single-crossing precision. Then:*

(i) Expected total investments:

a) There exists some $\hat{n} \geq k$ such that for all $n \geq \hat{n}$, expected total investments of workers are increasing in information precision.

b) If $n \leq k$, and the distribution of firms' types, F_Y, has an increasing hazard rate, then expected total investments of workers are decreasing in information precision.

(ii) Workers' expected welfare is increasing in information precision.

The result shows that, if workers are on the long side of the market and the number of workers is sufficiently large, then a more single-crossing precise signal always results in an increase of workers' expected total investments. In this case, only the high-ranked workers are matched in equilibrium. The expected types of these workers are increasing in the level of information and so are the externalities imposed on them. If the ratio of workers to firms is large enough only the effect on high-type workers matter. Consequently, the expected total

[20]To be precise, the expected externality imposed on the low posterior-type workers is non-increasing, since these workers may not be matched in equilibrium. Workers with zero investments do not adjust their investments.

[21]The results of Theorem 2 incorporate as special cases both, the comparative statics results on heterogeneity of Hoppe et al. (2009) and the results of Ganuza and Penalva (2010) on the expected valuation and the informational rent of the winning bidder, and the seller's expected revenue. More details on this are provided in Section 7.

investments, which capture the externalities workers impose on each other, will increase. Competition among workers may even be so strong that the expected investment of each individual worker is nondecreasing in information.[22]

By contrast, if there are more firms than workers, all workers are matched in equilibrium. We know that if the information level of workers increases, then the externalities imposed on the low-type workers decrease. If this effect drives the effect on expected total investments of workers, then they decrease as the information level of workers increases. This is the case if the distribution of firms' types has an *increasing hazard rate*, which implies that the externalities imposed on low-type workers have a higher impact on aggregate expected investments than those imposed on high-type workers (cf. Lemma 2 and Table 2).[23]

If workers hold private information, workers' expected welfare W_w captures the informational rents of workers. In the model that we consider, the matching mechanism is fixed. Hence, as expected, for a higher information level of workers, more information rent is left to the workers and the expected welfare of workers is increasing.

The next result establishes the effects of a higher information level of workers for the other side of the market, that is, on firms' expected total investments and welfare.

Theorem 3 (Firms' expected total investments and welfare)**.** *Let signal S_2 be more single-crossing precise than signal S_1.*

(i) Expected total investments of firms are increasing in information precision.

(ii) Firms' expected welfare:

- *a) There exists some $\hat{n} > k$ such that firms' expected welfare is increasing in information precision for all $n \geq \hat{n}$.*
- *b) If $n \leq k$ and F_Y has an increasing hazard rate, then firms' expected welfare is decreasing in information precision.*
- *c) If F_Y has a decreasing hazard rate, firms' expected welfare is always increasing in information precision.*

If workers have a higher information level, firms face a sample of potential match partners with a more heterogeneous distribution of posterior types. Consequently, the expected difference of the match outputs from being paired with one of two workers whose ranking differs only by one increases. Competition among firms increases, which results in firms increasing their expected investments. This is also true at the individual level – every firm will increase its expected investment.

[22]This result is easily established by combining Lemma 4 with (1).

[23]F_Y has an *increasing hazard rate* if $\frac{f_Y(y)}{1-F_Y(y)}$ is increasing in y. This property is a common assumption in mechanism design and satisfied by a large class of distribution functions, including the uniform, normal, and exponential distribution. For a detailed discussion see Bagnoli and Bergstrom (2005) and Ewerhart (2013).

Among the firms that are matched in equilibrium, the match output of high ranked firms is increasing in the information level of workers. For lower ranked firms it will typically be decreasing, unless there is a much larger number of workers than firms. Thus, lower ranked firms will be worse off if workers' hold more precise information whereas high ranked firms may profit. The effect on expected welfare of firms depends on which of these effects is dominant, and hinges on the distribution of firms' types and the sizes of the two sides of the market. If firms constitute the long side of the market and their distribution of types has an increasing failure rate, firms' expected welfare decreases the information level of workers increases. In this case, the increased competition among firms will eat up all of the additional match surplus made possible by the higher information level of workers.

Is it always welfare improving to provide workers with additional and hence more precise information? The answer is not obvious. For a higher information level of workers, there is a trade-off between a higher expected total match output and a possibly increase in (wasteful) investments. From the previous analysis we know that providing more information to workers always increases workers' expected welfare whereas firms' expected welfare may be decreasing. A higher information level of workers has a more direct effect on expected welfare of workers than on that of firms. This property may suggest that the first effect is stronger, which would imply that expected aggregate welfare is always increasing in the information level of workers.

However, this intuition is not correct. The following example shows that increasing the information level of workers may result in a decrease of aggregate welfare.

Example 3 (Expected aggregate welfare)**.**
Consider a matching market with three workers and three firms, $n = k = 3$. Workers' types are standard uniformly distributed, $X_i \overset{iid}{\sim} U\,[0,1]$, and the information technology is a truth-or-noise technology S_α with precision level α. In this setting, the posterior types of workers are uniformly distributed on $\left[\frac{1}{2}(1-\alpha),\, \frac{1}{2}(1+\alpha)\right]$, and the corresponding vector of posterior mean-order statistics is $(\mu^\alpha_{1:3}, \mu^\alpha_{2:3}, \mu^\alpha_{3:3}) = \left(\frac{1}{2} + \frac{1}{4}\alpha, \frac{1}{2}, \frac{1}{2} - \frac{1}{4}\alpha\right)$. From an ex-ante perspective, the expected posterior type of the highest worker is $\frac{1}{2} + \frac{1}{4}\alpha$. Suppose firms' types are represented by the vector $(\eta_{1:3}, \eta_{2:3}, \eta_{3:3}) = \left(\frac{2}{3}, \frac{1}{2}, \frac{1}{3}\right)$. Table 3 illustrates expected output, expected total investments and welfare of workers and firms for the given specifications.

In this setting, as the information level α increases, the externalities imposed on the lowest-ranked worker, $2 \cdot \left(\frac{1}{2} - \frac{1}{4}\alpha\right) \cdot \frac{1}{6}$, decrease whereas those imposed on the middle-ranked worker are constant. Consequently, workers' expected total investments, $T_w = \frac{1}{4} - \frac{1}{12}\alpha$, is decreasing in the information level α. Moreover, as the information level of workers increases, welfare of firms, $W_f = \frac{3}{4} - \frac{5}{24}\alpha$, decreases. In total, we obtain that the negative effect on

$O = \frac{3}{2} + \frac{1}{12}\alpha$	
$T_w = \frac{1}{4} - \frac{1}{12}\alpha$	$T_f = \frac{7}{24}\alpha$
$W_w = \frac{1}{2} + \frac{1}{6}\alpha$	$W_f = \frac{3}{4} - \frac{5}{24}\alpha$
$W = \frac{5}{4} - \frac{1}{24}\alpha$	

Table 3: Expected total output, investments and welfare for workers and firms, for $n = k = 3$, $X_i \overset{iid}{\sim} U[0,1]$, firms' types $(\eta_{1:3}, \eta_{2:3}, \eta_{3:3}) = \left(\frac{2}{3}, \frac{1}{2}, \frac{1}{3}\right)$, and a truth-or-noise technology of precision level α.

firms is stronger than the positive effect on workers:

$$\begin{aligned} \frac{\partial W}{\partial \alpha} &= \frac{\partial o}{\partial \alpha} + \frac{\partial W_f}{\partial \alpha} - \frac{\partial T_w}{\partial \alpha} \\ &= \frac{1}{12} - \frac{5}{24} + \frac{1}{12} = -\frac{1}{24} < 0. \end{aligned}$$

A notable feature of this example is, that even though workers' expected total investments are decreasing in the level of information, the negative effect on firms' welfare is so strong that expected aggregate welfare is decreasing. △

Remark This observation that increasing the information level of agents may not be welfare enhancing complements and strengthens existing results, which show that random matching may be welfare superior to assortative matching because it allows to avoid wasteful signaling or screening costs.[24] Random matching requires that neither side adopts separating strategies. In many settings this is unlikely to be true, be it because there is some information about a ranking of agents available in the market, or because it is simply infeasible.[25] In this case, our example shows that for a higher information level of workers, the increased competition among firms may be so strong that the increased investments of firms may eat up all gains from increased match output and decreased wasteful investments of workers. As a result, overall expected welfare may be decreasing in the information level of workers.

To better understand the informational effects on aggregate welfare, it helps to decompose aggregate welfare as $W = o + (W_f - T_w)$; the sum of total match output of workers, $o = \frac{1}{2}O$, and aggregate externalities imposed by workers on other agents, $W_f - T_w$. Here, T_w captures the aggregate externalities workers impose on each other, whereas W_f captures the aggregate externalities imposed on firms, i.e., agents on the other side of the market.

[24]See for example Hoppe et al. (2009), Condorelli (2012), and Chakravarty and Kaplan (2013).

[25]Of course, a way to implement the random matching is to ignore any investments of agents. However, if agents have some private information this is not a stable matching.

Thus, the effect of a higher level of information of workers on total welfare consists of the effect on workers' match output and the change in the aggregate externalities workers impose on all agents. By Proposition 1 we know that total match output is increasing in workers' information level whereas $W_f - T_w$ may be decreasing. In this case the effect on aggregate welfare depends on which of the two effects is dominant.

We conclude this section by identifying conditions, which each individually guarantee that aggregate welfare is increasing in workers' information level.

Theorem 4. *Aggregate welfare is increasing in information precision if one of the following conditions is satisfied:*
(i) F_Y *has a decreasing hazard rate, or*
(ii) $n < k$ *and* f_Y *is monotone decreasing.*[26]

6 Optimal Level of Precision

We now take the analysis one step further and assume that, in the information stage, the precision of the information technologies is not exogenously given, but can be chosen before the information stage. We call this game the *precision and matching game*.

In a precision and matching game, first, workers' information technology S_α is chosen from a set $\mathcal{S}$ of feasible information technologies, either collectively by one group of agents, or by a social planner. The information technology S_α is then implemented and the rest of the game proceeds as described in Section 2. In the information stage, every worker obtains a private signal from S_α, agents then update their beliefs according to Bayes' rule before they enter the matching stage. The timing in the precision and matching game is illustrated in Figure 3.

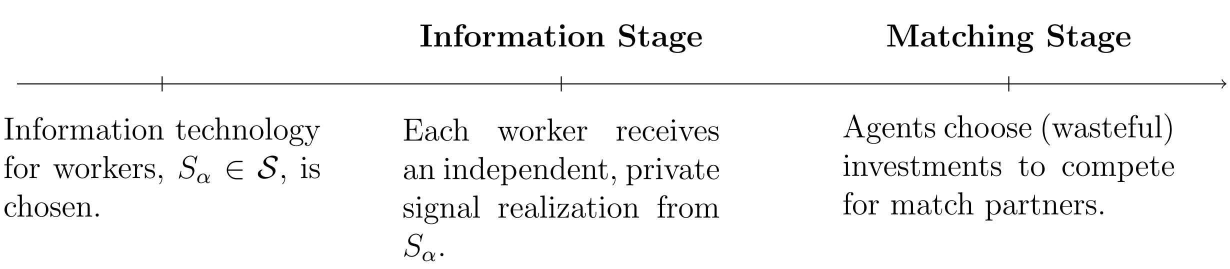


Figure 3: Timing in the *precision and matching game*.

[26] Every absolute continuous random variable with a decreasing hazard rate has a decreasing density function. But there exist also distributions with an increasing hazard rate and monotone decreasing density functions (cf. Bagnoli and Bergstrom, 2005).

We characterize the socially optimal information level, α^{so}, which maximizes aggregate welfare of all agents in the market, and compare it to the *worker-optimal* information level, α^{ao}, which maximizes workers' welfare. This is the optimal information level in a one-sided market in which only workers are active agents. It is also the information level that a designer who only cares about the well-being of workers would want to implement in a two-sided market.[27] Focusing on these two information levels allows to isolate the effects which originate from the two-sidedness of the matching market and are not prevalent in one-sided markets.

If information is costless it is easy to see from our previous discussion that the worker-optimal information level is to be perfectly informed (cf. Theorem 2). However, this is not necessarily the socially optimal information level since aggregate welfare may be decreasing in information precision (cf. Example 3). The same is true for the *firm-optimal* information level, i.e., the precision of workers' information which maximizes firms' welfare. In any case, if information is costless, the firm-optimal and the socially optimal level of information will always be extreme, that is, either full information or no information.

Costly Precision

We now consider the case when information is costly. To formally analyze this case, let $\mathcal{S}$ be a set of feasible information technologies which is totally ordered in terms of strict precision. That is, there exists some $\mathcal{A} \subseteq [0, \infty)$ such that $\mathcal{S} = \{S_\alpha\}_{\alpha \in \mathcal{A}}$ and S_α is more precise than $S_{\alpha'}$ if and only if $\alpha > \alpha'$. Information technology S_α is characterized by $\{G^\alpha(\cdot|x)\}_{x \in \mathcal{X}}$. For ease of presentation, we restrict attention to linear information models, with $E[X|S] = \alpha S + (1-\alpha)E[X]$, $\alpha \in [0, 1]$. The natural indexation in this case is to denote by S_α the information technology that results in $E[X|S] = \alpha S + (1-\alpha)E[X]$.

The following condition on the distribution of signals guarantees that for all precision levels $\alpha \in (0, 1)$, the distribution and density functions of the posterior estimates, H^α and h^α, are continuously differentiable in the precision level α:

Assumption 2 (Differentiable Signals)**.**
The marginal distribution of signal realizations G is twice continuously differentiable in s.

We assume that information costs have a 'pay per signal' structure. For information technology $S_\alpha \in \mathcal{S}$ of precision $\alpha \in [0, 1]$ every worker who receives a signal from S_α has to

[27] This applies to setting in which there is a lobby group representing agents on one side of the market. Examples include the parent empowerment movement or labor unions. It should be noted that workers would also choose α^{ao} if they could coordinate on a common information level. e.g. by collectively choosing an information technology.

pay $c(\alpha) \in \mathbb{R}^+$. Precision costs are increasing in α and capture for example investments in time or resources to generate or collect information. The precision-cost-function $c : [0,1] \to [0,\infty)$ is increasing and continuously differentiable with $c(0) = 0$ and $c'(0) = 0$. Since every worker obtains exactly one signal, total costs for information technology S_α are $C(\alpha) := nc(\alpha)$.

Given Assumption 2 expected welfare of workers and expected aggregate welfare are continuously differentiable in the level of information of workers (cf. Lemma 6). In order to identify and compare the worker-optimal and socially optimal information levels, we impose the following single-crossing conditions.

Assumption 3 (Single-crossing)**.**
(i) $\frac{\partial c/\partial \alpha}{\partial W_w/\partial \alpha}$ *is strictly increasing in* $\alpha \in (0,1)$.
(ii) $\frac{\partial c/\partial \alpha}{\partial W/\partial \alpha}$ *is strictly increasing in* $\alpha \in (0,1)$ *whenever* $\frac{\partial W}{\partial \alpha} > 0$.

For $\mathcal{S}$ being the set of truth-or-noise technologies and $X_i \overset{iid}{\sim} U[0,1]$, Assumption 3 is satisfied for convex precision costs.

We now characterize the relation between the worker-optimal and the socially optimal information level if information is costly.

Theorem 5. *In a precision and matching game, suppose Assumption 1–3 are satisfied.*

(i) The socially optimal information level of workers is higher than the worker-optimal level, $\alpha^{wo} \leq \alpha^{so}$, *if*
   a) workers constitute the long side of the market and n *is sufficiently large, or*
   b) the distribution of firms' types F_Y *has a decreasing hazard rate.*

(ii) The socially optimal information level of workers is lower than the worker-optimal level, $\alpha^{so} \leq \alpha^{wo}$, *if workers constitute the short side of the market and the distribution of firms' types has an increasing hazard rate.*

It is not surprising that in a precision and matching game the worker-optimal and socially optimal levels of precision do not coincide. Information of workers imposes an externality on firms. For a higher information level a better allocation can be achieved which increases total match output. However, more information also leads to more differentiation among workers, which increases competition among firms. This results in higher expected investments of firms. The relation between the worker-optimal and the socially optimal level of information depends on whether the overall effect of a higher information level of workers on firms is positive or negative.

Theorem 5 illustrates that in a relatively balanced market, with groups of similar sizes on each side of the market, the relation between the worker-optimal and socially optimal information level depends on the distribution of the informed agents' types. However, the socially optimal information level is always higher than the worker optimal information level, if the group of uniformed agents constitutes the long side of the market and is sufficiently large.

7 Applications

In this section we discuss implications of the results established in Section 5 and Section 6 for various market design settings, in particular auctions, contests and matching markets. In each of the applications we highlight certain features of our results and contributions.

7.1 Auctions

The standard private values auction setting is captured as a special case by our model. It corresponds to the case in which workers represent the bidders of the auction, and there is only one firm of a given, commonly known type, representing for example the auction platform or the object to be sold. The seller is a third party who collects the bids. As summarized in Table 4, the expected valuation of the winning bidder is $\frac{1}{2} \cdot O$, his expected information rent is W_w, and T_w represents the seller's expected revenue.

auctions	exp. valuation of the winning bidder	exp. revenue of the seller	exp. information rent of the winning bidder
general model	$\frac{1}{2}O = \mu^S_{1:n}$	$T_w = \mu^S_{2:n}$	$W_w = \mu^S_{1:n} - \mu^S_{2:n}$

Table 4: Translation of our results to the standard auction setting with n bidders.
Reminder: $\mu^S_{i:n}$ denotes the i^{th} mean order statistics of the posterior distribution of bidders' types.

Translating our results of Section 5 to the auction setting yields the following insights: Disclosing information to bidders increases the expected valuation of the winning bidder (Proposition 1) as well as his expected information rent (Theorem 2).[28] Moreover, the seller's expected revenue increases, if there are sufficiently many bidders (Theorem 2). If information disclosure is costly, the revenue maximizing level is below the efficient level (Theorem 5). These observations correspond to the results established in Ganuza and Penalva (2010) on information disclosure in auctions. Our results therefore include their results on information disclosure in auctions as a special case.

[28] For the result on expected information rents of the winning bidder a version of *strong precision* is needed. The result follows for both criteria of strong precision, the one used in this paper as well as the concept of *supermodular precision* adopted by Ganuza and Penalva (2010).

The methods from statistics that we adopt in this paper, provide an alternative and shorter way to prove these results. Using these statistical methods allows furthermore to strengthen the results of Ganuza and Penalva (2010). For example, the result on the expected informational rent of the winning bidder can be strengthened to the statement that providing more information to bidders increases the expected informational rent of the winning bidder in terms of first-order stochastic dominance and not only in expectation.[29]

The statistical methods used in this paper are powerful and should be explored further because they hold the promise to yield interesting results and insights in mechanism design settings with endogenous information. Let us support this point by providing a small, new result that we can establish by exploring these methods further.

Let $\mathcal{S} = \{S_\alpha\}_{\alpha \in \mathcal{A}}$ be a ordered set of information technologies such that $S_{\alpha'}$ is more precise than S_α, if $\alpha' > \alpha$. Say that the *information level* of bidder i is α_i, if he receives a signal from information technology S_{α_i} in the information stage. Bidders may receive signals from different information technologies, that is, of different precision, resulting in a profile of information levels of bidders $(\alpha_1, \ldots, \alpha_n)$. We say that the information level of bidders *weakly increases* if the information level of at least one bidder strictly increases, and the information levels of all other bidders are non-decreasing.

Proposition 2. *In a private values auctions setting consider any auction format that implements the efficient allocation. For any weak increase in the information levels of bidders, the expected value of the winning bidder increases.*

To our knowledge, this generalization of the results on information disclosure in auctions is new to the literature. It establishes that any weak increase in the information level of bidders will increase the expected efficiency of the allocation of the auction. For this result, the additional information provided to, or processed by, the individual bidders may be heterogeneous, which is natural feature in many situations.

Consider for example a seller who publicly discloses information that is relevant for bidders to learn about their valuation for the object for sale. Typically, the level of information that individual bidders extract from the publicly available data differs across bidders. Proposition 2 establishes that the effect on the expected valuation of the winning bidder does not depend on this detail. Providing more information will always increase the expected valuation of the winning bidder.

[29]The result is a simple corollary to theorem 3.B.31 in Shaked and Shanthikumar (2007). This was pointed out in a footnote in Ganuza and Penalva (2010) but the fact that this alternative proof yields a stronger result was mentioned only recently in Shaked et al. (2012).

7.2 Two-sided Matching Markets

When applied to matching markets, our results provide a first theoretical study of the effects of private information and the information level of market participants on the equilibrium outcome in matching markets.

As we have discussed in Section 4, a higher information level of workers leads to more differentiation among workers in the matching game. This effect allows for a better allocation: high-abilitiy workers are more likely to be matched to a firm of similar ranking, which results in an increase of total match output (Proposition 1).[30]

Moreover, a higher information level and the resulting higher differentiation among workers, also raises the stakes for the firms of being matched to a better or worse partner. This results in an increase of firms' expected signaling investments (Theorem 3).

The effect on workers' expected signaling investments is less clear-cut and depends on certain features of the market (Theorem 2). For a higher information level of workers, the (marginal) benefit from obtaining a better match increases for high-ranked workers, whereas it is decreasing for low-ranked workers. This results in high-ranked workers increasing their investments in signaling whereas lower ranked workers may invest less. If workers constitute the long side of the market, only high-ranked workers are matched in equilibrium and workers' expected total investments are increasing. If there are more firms than workers, the effect may be reveres.

These results illustrate, that in finite matching markets, some of the effects of a higher information level of market participants depend on whether information is disclosed to agents on the short or the long side of the market. This new insight is made possible because our comparative statics result in Section 5 apply for arbitrary finite group sizes on the two sides of the market.[31] This observation highlights that it is important to study matching market models with a finite number of agents and not restrict attention solely on the case with a continuum of agents on both sides of the market.[32]

An important insight from our analysis is the following: In a two-sided matching market

[30] This effect is observed in empirical studies. For example, in their study Hoxby and Turner (2013) provide a subgroup of high-school seniors with additional information about their college opportunities and find that, for students who received information, the probability to enroll in a college that matches their abilities increases significantly.

[31] This also allows us to consider information disclosure in auctions as a special case of our results. Moreover, the projection of our results to the model of Hoppe et al. (2009) generalizes their results on comparative statics effects of group heterogeneity (they only consider the case $n = k$).

[32] Most models which discuss comparative static settings study models with a continuum of agents on each side of the market. Considering a continuum of agents is often a reasonable and very useful assumption, since it avoids the technicalities of having to deal with order statistics. However, the point we want to make here is that it is also important to study the model with finite sets of agents.

in which both sides of the market invest in wasteful signaling to compete for match partners, the trade-off between a better allocation and a potential increase in wasteful signaling investments may result in a decrease of expected aggregate welfare when the information level of market participants increases (Example 3). This feature is specific to two-sided markets. Welfare of agents on the side of the market receiving more information is always increasing in expectation (Theorem 2). Consequently, in settings in which only one side of the market are active agents, providing these agents with additional information would unambiguously increase expected welfare. By contrast, in a two-sided matching market, for an increase in the information level of agents, the amplified competition among agents within their groups and the resulting increase in signaling investments may eat up all additional match surplus.

What can we learn from this? Our results indicate that in a two-sided matching market more information is not necessarily better if the objective is to maximize overall expected welfare. However, there are often different or additional objectives like equalizing the information level across agents, fairness considerations or incentivizing schools or colleges to invest in their quality. Our results yield insights into the last aspect. If parents are provided with more information about school choices, the highest-ranked schools profit most from informed choices of parents, whereas low-ranked schools may be worse off (Theorem 3).[33] This may serve as a formal rationale for the claim often raised by the parent empowerment movement, that providing parents with more information results in them making more informed choices, which – in the long run – will increase school quality. From this perspective, it may even be good to let a lobby for one side of the market, determine the level of information of market participants, even though they do not fully interalize the costs and benefits from additional information and thus will not choose the socially optimal information level.[34] Theorem 5 suggest that, if the uninformed agents – the workers – constitute the long side of the market, the worker-optimal level of information for this side of the market is higher than the socially optimal level. Given that only high-quality firms profit from the informed choices by workers, this incentivizes firms to compete for the highest-rank among their peers which may induce them to invest in quality-enhancing policies.

[33] This claim still remains true if schools are not considered to be active agents and therefore do not invest in signaling about their types.

[34] Distributing information among parents, providing more or less detailed information on websites, or determining the precision level of standardized test like the SAT are examples of technologies that can serve to influence the level of information of market participants.

7.3 Contests

Our model can also includes a rank-order tournament or contest setting as a special case. To see this, interpret agents on one side of the market, say the firms, as representing the prizes in a promotion tournament, with commonly known values $\eta_{1:k} \geq \cdots \geq \eta_{k:k}$. This side of the market is passive. Workers represent the participants of the contest. Workers' types reflect their abilities, and their investments correspond to the effort, which they exert. With this interpretation, workers' investments are not wasteful but they are collected by a third party – the company or organization running the promotion tournament.

In promotion tournaments there are two natural objectives: To promote the best workers and to maximize workers' efforts. Translating our results from Section 5 and Section 6 to the promotion contest, we obtain the following predictions. More information of competitors increases the probability to promote the best workers (Proposition 1). If the ratio of workers to prizes is sufficiently large, then workers' expected overall effort is increasing in their information level. However, if this ratio is too small, providing information to the workers may not be effort enhancing – not even on an aggregate level (Theorem 2).

There is a second option to project our model to a contest, interpreting firms as contestants and the mean-order statistics of posterior types of workers, $\mu^S_{1:n} \geq \cdots \geq \mu^S_{n:n}$, as prizes. In this case, the translation of Theorem 3 yields the well-known observation that increasing the prize-spread in contests results in an increase in workers' effort.[35]

A typical question in the contest literature is how to design an optimal contest in order to maximize workers' effort. Our results indicate that information management through feedback systems may serve as an useful element of contest design. Let us provide some details for this insight. The standard design element that is usually considered in the contest literature are prizes. The number and distribution of prizes in a contest affect workers' effort, and can therefore be used to design an optimal prize-structure that maximizes workers' efforts. However, in some organizations it may not be feasible to implement the optimal prize-structure suggested by theoretical models, because there are certain constraints on the number or distribution of prizes. For example, in promotion tournaments the prize-structure is determined by the wage schedule and the number of positions on each level of the organization. In situations in which the optimal prize-structure cannot be implemented, a designer could influence workers' effort by implementing a feedback systems to manipulate the information level available to workers. We refer to this design element as *information management*.

[35]See for example Lazear and Rosen (1981), Moldovanu et al. (2007), Connelly et al. (2014) and references therein – also of empirical studies supporting these theoretical predictions.

Our results shed light on properties of optimal feedback systems in contests. They suggest that we should observe different feedback systems depending on the ratio of workers and prizes in a contest. In organizations with steeper hierarchies or an up-or-out system, we should expect stronger feedback systems to be in action, for example a high frequency of periodical performance reports. By contrast, for organizations with flat hierarchies or *promotion by seniority* practices, our results predict less sophisticated feedback structures. These predictions seem to be in line with common practices. For example, large consulting firms with an up-or-out policy are known to have a very rigorous feedback structure.

8 Related literature

This paper is related to various strands of literature. It is connected to the vast matching literature that emerged from the seminal papers by Gale and Shapley (1962), Shapley and Shubik (1971) and Becker (1973). Most of the theoretical analysis of matching markets focuses on complete information models in which agents' preferences, types, and match values are common knowledge. There is an emerging literature studying incomplete information matching models and issues of screening and signaling which arise therein. See for example Hoppe et al. (2009), Hopkins (2012), and Bilancini and Boncinelli (2013). Our second-stage game is based on the models analyzed in Hoppe et al. (2009) and Hopkins (2012). They study two-sided matching markets, in which agents on one or both sides of the market have private information about their characteristics. Agents invest in costly signaling à la Spence (1973) to compete for match partners. As Hoppe et al. (2009) we consider a small market with a finite number of agents, whereas Hopkins (2012) studies a model with a continuum of agents. In a related paper by Bilancini and Boncinelli (2013), agents on both sides of the market have private information about their skills and can choose whether or not to disclose this information. For one side of the market information is not verifiable and disclosing information yields certification costs. All of the aforementioned papers consider matching markets in which agents have private information about their characteristics, and analyze the costs and benefits from disclosing this information. By contrast, our focus is on disclosing information to agents about their types. We study how different information levels of participants in (matching) markets affect the resulting equilibrium properties and welfare.

Related questions are addressed in the literature on information disclosure in auctions. Ganuza and Penalva (2010) discuss the effects of different information levels of buyers in a second-price auction, whereas Bergemann and Pesendorfer (2007), Esö and Szentes (2007), and Ganuza and Penalva (2014) adopt a mechanism design perspective. In a private values environment, these papers discuss the revenue maximizing information structure and selling mechanism for the seller. Similar to Ganuza and Penalva (2010) we focus on a given mechanism and study how different information levels affect the equilibrium.

Our analysis extends the discussion of information disclosure in auctions to two-sided matching markets and can also be applied to contests and rank-order tournaments. A recent survey of the contest literature is provided by Connelly et al. (2014). The focus of most of these papers, for example Moldovanu and Sela (2001, 2006) is on optimal contest design, that is, on the optimal portfolio of prizes or how to split the contestants in subgroups to achieve the designer's objective. There is a growing literature studying the role of feedback and optimal feedback systems in contests. Examples include Aoyagi (2010), Goltsman and Mukherjee (2011), Hansen (2013), and Ederer (2010). All of these models restrict attention to the two-agent case and most of them only consider full or no disclosure policies. By contrast, we allow for arbitrary finite numbers of prizes and workers. A new insight that can be gained from our results is that the optimal feedback policy depends on the ratio of workers to prizes (see discussion in Subsection 7.3).

This paper also ties to the literature that identifies and explores connections between auctions and matching markets in order to establish new results. See for example Demange and Gale (1985) and Hatfield and Milgrom (2005). The matching market that we study, can be considered as the combination of two multi-object auctions with two sides of active agents (cf. discussion in Section 3). To establish our results we use this connection between matching markets and auctions and identify a relation between the statistical methods used by Hoppe et al. (2009) and the type of precision criterion introduced by Ganuza and Penalva (2010).[36]

Our paper also relates to the growing literature on pre-match investments in matching tournaments. Examples include Cole et al. (2001), Peters and Siow (2002), and Mailath et al. (2013) and Dizdar (2015). Pre-match investments generate first-order effects on agents' types. Similarly, Hopkins (2012) studies such first-order effects, interpreting shifts in agents' type distribution in terms of first-order stochastic dominance as a more competitive environment. By contrast, in our analysis, investments in information yield second-order effects. A higher level of information leads to a more dispersed distribution of workers' posterior types in the second-stage matching game.

[36]Methodologically, a related paper is Chi (2014), who uses statistical methods to study informational effects in Bayesian decision problems.

9 Conclusion

In this paper we studied the impact of the level of information available to market participants in a two-sided matching market. We illustrated that for a higher information level of workers there is a trade-off between the increased match surplus from the better allocation, and the welfare reducing effects of increased competition among agents. It was shown that the increased competition among agents may be so strong that it eats up all additional match surplus. In this case, a higher information level of market participant reduces welfare. Our results not only provide a first study of information disclosure in matching markets, but can also be applied beyond the matching setting. We discussed implications of our results for auctions and contests.

A notable distinction between the effects of information disclosure in auctions and matching markets is the following: In an auction, a seller faces a trade-off between efficiency and having to leave information rents to the buyers. In a matching market information disclosure yields a trade-off between allocative efficiency and the welfare-reducing effects of increased competition among agents on both sides of the market.

The setup of the model and the discussion of applications to different market design settings illustrated how these settings are connected. We used these insights in this paper to identify a relation between the discussions in Hoppe et al. (2009) and Ganuza and Penalva (2010). Establishing a link between the methods adopted in these papers provided us with a new approach to study the impact of information disclosure in two-sided matching markets and related applications. We believe that the connections that we have identified between the different models and concepts will prove to be useful in future research, in particular to study mechanism design problems with endogenous information of agents.

Appendix

A Technical Prerequisites

In this section we present the main techniques used to prove our results. The methods stem from statistics and reliability theory. Shaked and Shanthikumar (2007) provide a comprehensive treatment of order statistics whereas Marshall et al. (2011) is a good reference for the theory of majorization. If not indicated otherwise, all definitions and theorems stated in this section can be found in these two books.

The single-crossing property of quantile functions that we use in Definition 1 is equivalent to the distribution of posterior estimates being ordered in terms of the star order (see Shaked and Shanthikumar (2007), Section 4.B).

Fact 1. If S_2 is more single-crossing precise than S_1, then $E[X|S_2]$ is greater than $E[X|S_1]$ in the *star-order*, $E[X|S_2] \geq_* E[X|S_1]$.

Notation. $X \leq_{MPS} Y$, then Y is a mean-preserving spread of X.
$X \leq_* Y$, then Y is greater in the star order than X.

Definition 2. Consider two ordered n-dimensional real-valued vectors $\mathbf{a} = (a_1, \ldots, a_n)$, and $\mathbf{b} = (b_1, \ldots, b_n) \in \mathbb{R}^n$, with $a_1 \geq \cdots \geq a_n$ and $b_1 \geq \ldots b_n$. We say that $\mathbf{a}$ *submajorizes* $\mathbf{b}$, ($\mathbf{a} \succ_{sub} \mathbf{b}$), if

$$\sum_{i=1}^{m} a_i \geq \sum_{i=1}^{m} b_i \quad \text{for all } m = 1, \ldots, n. \tag{2}$$

If in addition (2) holds with equality for $m = n$ we say that $\mathbf{a}$ *majorizes* $\mathbf{b}$, ($\mathbf{a} \succ \mathbf{b}$).

A function $\phi : \mathbb{R}^n \supseteq A \longrightarrow \mathbb{R}$ is *Schur-convex* (resp. *Schur-concave*) if, whenever a majorizes b, $\mathbf{a} \succ \mathbf{b}$, then $\phi(\mathbf{a}) \geq \phi(\mathbf{b})$ (resp. $\phi(\mathbf{a}) \leq \phi(\mathbf{b})$).
If $\mathbf{a} \succ_{sub} \mathbf{b}$ then $\phi(\mathbf{a}) \geq \phi(\mathbf{b})$ for every Schur-convex and increasing function ϕ.

To proof our results we repeatedly use the following important results from statistics.[37]

Theorem 6 (Cal and Carcamo 2006)**.** *Let X and Y be integrable random variables with equal means and $F(0) = G(0) = 0$. Then if $X \leq_{MPS} Y$, the vector of mean order statistics of Y, $(E[Y_{1:n}], \ldots, E[Y_{n:n}])$* majorizes *the vector of mean order statistics of X for all $n \geq 1$. That is,*

$$(E[Y_{1:n}], \ldots, E[Y_{n:n}]) \succ (E[X_{1:n}], \ldots, E[X_{n:n}]).$$

Theorem 7 (Barlow and Proschan (1966))**.** *Let X and Y be integrable random variables with equal means and $F(0) = G(0) = 0$. Then if $X \leq_* Y$ this implies $X \leq_{MPS} Y$, and moreover*

(i) For $1 \leq r \leq n$:

$$\sum_{i=r}^{n} i \cdot (E[X_{i:n}] - E[X_{i+1:n}]) \geq \sum_{i=r}^{n} i \cdot (E[Y_{i:n}] - E[Y_{i+1:n}]).$$

(ii) For $a_1 \leq \cdots \leq a_n$:

$$\sum_{i=1}^{n} a_i \cdot i \cdot (E[X_{i:n}] - E[X_{i+1:n}]) \geq \sum_{i=1}^{n} a_i \cdot i \cdot (E[Y_{i:n}] - E[Y_{i+1:n}]).$$

[37] In their paper Cal and Carcamo (2006) establish this result for random variables ordered in terms of the convex-order. In our informational setting we always compare random variables with finite and equal means. In this case the convex order is equivalent to a mean-preserving spread.

Lemma 2. *Let F be a distribution function with $F(0) = 0$ and an increasing hazard rate (IHR). Then, for fixed n, the normalized spacings of order statistics $i \cdot (X_{i:n} - X_{i+1:n})$ are stochastically increasing in $i = 1, \ldots, n$. That is:*

$$(X_{1:n} - X_{2:n}) \leq_{FOSD} 2 \cdot (X_{2:n} - X_{3:n}) \leq_{FOSD} \cdots \leq_{FOSD} n \cdot (X_{n:n} - X_{n+1:n}) .$$

If F has a decreasing hazard rate (DHR), then the normalized spacings are stochastically decreasing in i.

We will also need the following result

Lemma 3. *Let X, Y be nonnegative random variables with distribution functions F and G, respectively, such that $F(0) = G(0) = 0$. If $X \leq_* Y$ then*
(i) $\frac{E[Y_{i:n}]}{E[X_{i:n}]}$ *is decreasing in i, and*
(ii) $\frac{E[Y_{i:n}]}{E[X_{i:n}]}$ *is increasing in n.*

B Proofs

Proof of Lemma 1. Is a direct corollary of theorem 3.6 in Barlow and Proschan (1966). □

Proof of Proposition 1. If $S_2 \succsim S_1$ then $E[X|S_2] \geq_{MPS} E[X|S_1]$ and, by Theorem 6, $(\mu^{S_2}_{1:n}, \ldots, \mu^{S_2}_{n:n}) \succ (\mu^{S_1}_{1:n}, \ldots, \mu^{S_1}_{n:n})$.

For $k \geq n$, $O = \sum_{i=1}^{n} \eta_{i:k}\mu_{i:n}$ is Schur-convex in the vector of mean order statistics of workers' characteristics and consequently if $S_2 \succsim S_1$ then $O(S_2) \geq O(S_1)$.

For $k < n$, $O = \sum_{i=1}^{k} \eta_{i:k}\mu_{i:n}$ is Schur-convex in the *truncated* vector of mean-order statistics of workers' posterior types, $\mu|_{\leq k} = (\mu_{1:n}, \ldots, \mu_{k:n})$. A higher information level of workers only results in (weak) submajorization of the truncated vectors of mean-order statistics, i.e.

$$S_2 \succsim S_1 \quad \Rightarrow \quad \mu^{S_2}\big|_{\leq k} \succ_{sub} \mu^{S_1}\big|_{\leq k} .$$

Since O is increasing and Schur-convex it follows that $O(S_2) \geq O(S_1)$. □

In order to prove Theorem 2 we first establish a technical Lemma. To state and prove it we need the following fact.

Fact 2 (Theorem 3.A.5 in Shaked and Shanthikumar 2007)**.** The following conditions are each sufficient and necessary for $X \leq_{MPS} Y$

$$\int_0^p \left(G^{-1}(u) - F^{-1}(u)\right) \mathrm{d}\, u \leq 0 \quad \forall\, p \in [0,\, 1]\,, \quad \text{and} \tag{3}$$

$$\int_p^1 \left(G^{-1}(u) - F^{-1}(u)\right) \mathrm{d}\, u \geq 0 \quad \forall\, p \in [0,\, 1]\,. \tag{4}$$

Lemma 4. *Let X, Y be random variables with continuous differentiable distributions F and G and equal means, such that $X \leq_{MPS} Y$. Then, for every $k \in \mathbb{N}$ there exists some $\hat{n}_k$ such that*

$$E\left[X_{k:n}\right] \leq E\left[Y_{k:n}\right] \quad \forall\, n \geq \hat{n}_k.$$

Proof. The methods in this proof are similar to the ideas used to prove theorem 1 in Ganuza and Penalva (2010).
We can apply the probability integral transformation[38] to obtain the following simple formula for the k^{th} order statistics of X:

$$E\left[X_{k:n}\right] = \frac{n!}{(k-1)!(n-k)!} \int_0^1 F^{-1}(u) u^{n-k} (1-u)^{k-1} \mathrm{d}\, u$$

Set $\phi(u) := G^{-1}(u) - F^{-1}(u)$. Since G and F are continuously differentiable, by the inverse function theorem F^{-1} and G^{-1} are continuous and so is ϕ.

Suppose that $\phi(u) \neq 0$ on a subset of $[0,1]$ with nonempty interior.[39] Define $L := \{u \in X\,[0,1] : \phi(u) < 0\}$ and $\overline{u} := \sup\{L\}$. (3) and (4), continuity of ϕ and the assumption that $\phi(u) \neq 0$ on a subset of $[0,1]$ of positive measure imply that $\overline{u} \in (0,\, 1)$. We obtain that there exist $p_1,\, p_2 \in (\overline{u},\, 1]$ such that $\phi(u) > 0$ for all $u \in [p_1,\, p_2]$. Set $c_1 := \min_{u \in [0, p_1]}\{\phi(u)(1-u)^{k-1}\}$ and $c_2 := \min_{u \in [p_2, 1]}\{\phi(u)(1-u)^{k-1}\}$. By construction $c_1 < 0$ and $c_2 > 0$. This yields:

$$\begin{aligned} E\left[Y_{k:n}\right] - E\left[X_{k:n}\right] =& k\binom{n}{k} \int_0^1 \left(G^{-1}(u) - F^{-1}(u)\right) u^{n-k} (1-u)^{k-1} \mathrm{d}\, u \\ \geq& \frac{n!}{(k-1)!(n-k+1)!} p_2^{n-k+1} \left[\left(\frac{p_1}{p_2}\right)^{n-k+1} (c_1 - c_2) + c_2\right] \end{aligned}$$

Set $\hat{n} := \lceil k - 1 + \frac{\ln\left(\frac{c_2}{c_2 - c_1}\right)}{\ln\left(\frac{p_1}{p_2}\right)} \rceil$ where $\lceil x \rceil$ denotes the smallest natural number greater or equal

[38]For every random variable X with continuous c.d.f. F and density f, the transformed random variable $F(X)$ has a standard uniform distribution, $F(X) \sim U\,[0,1]$

[39]The case $\phi(u) = 0$ a.e. is trivial.

than x. It follows that:

$$\frac{n!}{(k-1)!(n-k+1)!}p_2^{n-k+1}\left[\left(\frac{p_1}{p_2}\right)^{n-k+1}(c_1-c_2)+c_2\right]\geq 0 \quad \forall\, n\geq \hat{n}$$

□

Proof of Theorem 2.

(i) *Total expected investments:*

a) By Lemma 4 there exists some $\hat{n}>k+1$ such that for every $n\geq\hat{n}$, $\mu^{S_2}_{k+1:n}-\mu^{S_1}_{k+1:n}\geq 0$. Since $\mu^{S_1}_{k+1:n}\geq 0$, it follows that

$$\frac{\mu^{S_2}_{k+1:n}}{\mu^{S_1}_{k+1:n}}\geq 1 \quad \forall n\geq\hat{n}.$$

By Lemma 3, $\frac{\mu^{S_2}_{i:n}}{\mu^{S_1}_{i:n}}$ is decreasing in i for every n and it follows that $\mu^{S_2}_{i:n}-\mu^{S_1}_{i:n}\geq 0$ for all $i\leq k+1$. We obtain that for all $n\geq\hat{n}>k$:

$$\begin{aligned} T_w(S_2)-T_w(S_1) &= \sum_{i=1}^{\min\{n,k\}}(\eta_{i:k}-\eta_{i+1:k})\cdot\left(\mu^{S_2}_{i+1:n}-\mu^{S_1}_{i+1:n}\right)\\ &=\sum_{i=1}^{k}\underbrace{(\eta_{i:k}-\eta_{i+1:k})}_{>0}\cdot\underbrace{\left(\mu^{S_2}_{i+1:n}-\mu^{S_1}_{i+1:n}\right)}_{\geq 0}\geq 0. \end{aligned}$$

b) If F_Y has an increasing hazard rate, then the normalized spacings $i(\eta_{i:k}-\eta_{i+1:k})$ are stochastically increasing in i (Lemma 2). Set $\widetilde{T}_w := \sum_{i=0}^{n} i(\eta_{i:k}-\eta_{i+1:k})\mu_{i+1:n}$. Then, for $n\leq k$, $T_w=\widetilde{T}_w$ and $\widetilde{T}_w$ is Schur-concave in the vector of mean order statistics of workers' characteristics. It follows that T_w is decreasing (non-increasing) in the level of information of workers.

(ii) *Workers' expected welfare:*

Set $a_i := -\eta_{i:k}$. Then, applying Theorem 7 (ii) yields

$$S_2 \succsim_* S_1 \quad\Rightarrow\quad W_w(S_2)\geq W_w(S_1).$$

□

Proof of Theorem 3.

(i) *Total expected investments:*

Analogous to the proof of Theorem 2 (ii) whereas the case-by-case analysis is now for $k\leq n$

and $k > n$.

(ii) *Firms' expected welfare:*
If F_Y has a decreasing hazard rate, by Lemma 2 the normalized spacings $i(\eta_{i:k} - \eta_{i+1:k})$ are stochastically decreasing in i. Consequently, W_f is Schur-convex in the vector of conditional mean order statistics of workers. By Theorem 6 it follows that $W_f(S_2) \geq W_f(S_1)$, if S_2 is more precise than S_1. The results for the case when F_Y has an increasing hazard rate, follow from arguments analogous to those used to prove Theorem 2 (i). □

Proof of Theorem 4. $W = W_w + W_f$. Rearranging terms yields:

$$W(S) = \underbrace{\left[\sum_{i=1}^{\min\{n,k\}} i \cdot \left(\mu^S_{i:n} - \mu^S_{i+1:n}\right)(\eta_{i:k} - \eta_{i+1:k})\right]}_{(W_f - T_w)(S)} + \underbrace{\sum_{i=1}^{\min\{n,k\}} \mu^S_{i:n}\eta_{i:k}}_{0.5O}$$

By Proposition 1 we know that total match output is increasing in precision. Whether aggregate welfare is increasing or decreasing in the level of workers' information depends on the effect on $W_f - T_w$, and, for the case that $W_f - T_w$ is decreasing, on which of these effects dominates. Throughout the proof, let S_2 be more single-crossing precise than S_1.

(i) If F_Y has a decreasing hazard rate, both W_w and W_f are increasing in precision (cf. Theorem 2 and Theorem 3) and so is $W = W_w + W_f$.

(ii) Suppose $n < k$ and f_Y is monotone decreasing. In this case

$$W = \sum_{i=1}^{n} \mu^S_{i:n}\eta_{i:k} + \left[\sum_{i=1}^{n} i \cdot \left(\mu^S_{i:n} - \mu^S_{i+1:n}\right)(\eta_{i:k} - \eta_{i+1:k})\right].$$

We use the following result which establishes that the spacings of order statistics from random variables with monotone density functions can be ordered in terms of stochastic dominance.[40]

Lemma 5. *Let $Y_1, \ldots, Y_n$ be independently, identically distributed random variables with finite support and density function f_Y. Then,*

(i) if f_Y is monotone increasing (non-decreasing)

$$Y_{i:n} - Y_{i+1:n} \leq_{FOSD} Y_{i+1:n} - Y_{i+2:n} \quad \forall\, i = 1, \ldots, n-2$$

[40]This result can be found in Shaked and Shanthikumar (2007).

(ii) if f_Y is monotone decreasing (non-increasing)

$$Y_{i:n} - Y_{i+1:n} \geq_{FOSD} Y_{i+1:n} - Y_{i+2:n} \quad \forall\, i = 1, \ldots, n-2$$

It follows directly that if f_Y is monotone decreasing, the expected spacings of mean order statistics $(\eta_{i:k} - \eta_{i+1:k})$, $i = 1, \ldots, k-1$ are decreasing in i. Setting $a_i := -(\eta_{i:k} - \eta_{i+1:k})$, by Theorem 7 (ii) we obtain $(W_f - T_w)(S_2) \geq (W_f - T_w)(S_1)$, for $S_2 \succsim_* S_1$. It follows that aggregate welfare is increasing in workers' information level. □

Lemma 6. *For linear information technologies, under Assumption 2, for all $\alpha \in (0,1)$, H^α and h^α are continuously differentiable in the precision level α. Moreover, O, T_w, T_f, W_w, W_f are continuously differentiable in $\alpha \in (0,1)$.*

Proof. For $\alpha \neq 0$, set $\phi(\alpha, w) := \frac{w-(1-\alpha)E(X)}{\alpha}$. Then, for linear information technologies and $\alpha \neq 0$, $H^\alpha(w) = G(\phi(\alpha, w))$, and $h^\alpha(w) = \frac{1}{\alpha} g(\phi(\alpha, w))$, for $\alpha = 0$, $H = G$. By Assumption 2 and since $\phi(w, \alpha)$ is continuously differentiable in $\alpha \in (0,1)$, $H^\alpha(w)$ and $h^\alpha(w)$ are continuously differentiable in α. Moreover, if H^α and h^α are continuously differentiable in α then so are the distributions of order statistics $H^\alpha_{i:n}$. The densities $h^\alpha_{i:n}$ are continuous in α for all $i = 1, \ldots, n$. This implies that the conditional mean order statistics $E\left[\widehat{X}^\alpha_{i:n}\right]$ are continuously differentiable in α. It follows that W, W_w, T_f, T_w, O are continuously differentiable in α. □

Proof of Theorem 5. The marginal value of information for workers is $\frac{\partial W_w}{\partial \alpha}$ and the socially marginal value is $\frac{\partial W}{\partial \alpha} = \frac{\partial W_w}{\partial \alpha} + \frac{\partial W_f}{\partial \alpha}$. By Theorem 3, if F_Y is DHR or if $n \geq \hat{n} > k$ then $\frac{\partial W_f}{\partial \alpha} > 0$ and it follows that, at any information level $\alpha \in (0,1)$, $\frac{\partial W}{\partial \alpha} > \frac{\partial W_w}{\partial \alpha}$. However, if F_Y is IHR and $n \leq k$, then social marginal gains from higher precision are lower than the marginal gains for workers, $\frac{\partial W}{\partial \alpha} < \frac{\partial W_w}{\partial \alpha}$.

In the *precision and matching game* with costly precision, the optimization problem for workers is:

$$\max_{\alpha \in [0,1]} \{U_c(\alpha) = W_w(S_\alpha) - n \cdot c(\alpha)\}$$

and for the social planner:

$$\max_{\alpha \in [0,1]} \{U_{SP}(\alpha) = W(S_\alpha) - n \cdot c(\alpha)\}$$

Given our assumptions on the cost function and Assumption 2, U_c and U_{SP} are continuously differentiable in α. By the extreme value theorem this guarantees the existence of a solution to the optimization problem of workers, respectively the social planner. The single-crossing conditions, **(SC)** and **(SC$_\mathbf{C}$)**, establish uniqueness.

If U_c is increasing on $[0,1]$, then the optimal level of precision for workers is $\alpha^{wo} = 1$, otherwise it is characterized by:

$$\left.\frac{\partial W_w}{\partial \alpha}\right|_{\alpha=\alpha^{wo}} = n \left.\frac{\partial c}{\partial \alpha}\right|_{\alpha=\alpha^{wo}} \tag{I}$$

Analogous reasoning shows that the unique socially optimal level of precision is either $\alpha^{so} \in \{0,1\}$ or an interior solution exists which is characterized by:

$$\left.\frac{\partial W}{\partial \alpha}\right|_{\alpha=\alpha^{so}} = n \cdot \left.\frac{\partial c}{\partial \alpha}\right|_{\alpha=\alpha^{so}} \tag{II}$$

Suppose that F_Y is DHR or that $n \geq \hat{n} > k$. In this case, at any information level $\widetilde{\alpha}$, the marginal gains for workers from higher precision are lower than the social marginal gains, $\left.\frac{\partial W_w}{\partial \alpha}\right|_{\alpha=\widetilde{\alpha}} < \left.\frac{\partial W}{\partial \alpha}\right|_{\alpha=\widetilde{\alpha}}$. Given uniqueness of the worker-optimal and the socially optimal level of precision we obtain $\alpha^{wo} \leq \alpha^{so}$.

The result for F_Y being IHR and $n < k$ follows by analogous reasoning. □

Proof of Proposition 2. This result is a direct corollary of Theorem 7.6 in Chapter 4, Barlow and Proschan (1981). □

C Discussion and relation to other informativeness criteria

Given our assumption that signals are monotone the natural informativeness criterion to use is the concept of *effectiveness* introduced by Lehmann (1988).[41] The basic idea behind this concept is that for a given state space X, information technology S_2 is more informative about X than S_1, if the conditional distribution of S_2 is more dependent on X than that of S_1. Formally,

Definition 3 (Effectiveness, Lehmann (1988))**.** Given X, let S_1 and S_2 be two signals which satisfy the Assumption 1. Then S_2 is said to be *more effective* than S_1 if for all s

$$G_{S_2}^{-1}(G_{S_1}(s|x)|x) \quad \text{is nondecreasing in } x.$$

Mizuno (2006) shows that for a more effective signal about X the resulting distribution of conditional expectations is more dispersed.

[41] Persico (2000) refers to this concept as *accuracy*. Effectiveness applies to monotone decision problems and requires less restrictive conditions than *sufficiency* (Blackwell, 1951) to compare signals in terms of their informativeness.

Theorem 8 (Mizuno 2006). *If signals are monotone, then if S_2 is more effective than S_1, it follows that S_2 is more integral precise than S_1 for all priors.*

Our definition of precision is similar to the notion of *integral* and *supermodular precision* in Ganuza and Penalva (2010) but the stochastic orders used to define these concepts differ. Our *precision* criterion in Definition 1 is based on the star order whereas *integral precision* is based on the *convex order* and *supermodular precision* is based on the *dispersive order*.[42] We briefly discuss the relation of these criteria which amounts to analyzing the relation of the stochastic orders.[43]

Let X and Y be two random variables with interval support and distribution functions F and G, respectively. We write $X \leq_* Y$ for X being smaller than Y in the star order, and $X \leq_{cx} Y$ and $X \leq_{disp} Y$ for X and Y ordered in terms of the convex, respectively dispersive order.

For a random variable X and signals S_1 and S_2, by the law of iterated expectations $E\left[E\left[X|S_1\right]\right] = E\left[E\left[X|S_1\right]\right] = \mu$. Consequently, in our informational setting we always compare random variables with finite and equal means. In this case the *convex order* is equivalent to the concept of second order stochastic dominance. Moreover, the dispersive order and the star order are both stronger than the convex order, that is

$$\begin{aligned} X \leq_{disp} Y \quad &\Rightarrow \quad X \leq_{cx} Y \text{ and} \\ X \leq_* Y \quad &\Rightarrow \quad X \leq_{cx} Y. \end{aligned}$$

For the star order and the dispersive order the following relation holds:

$$X \leq_* Y \quad \Leftrightarrow \quad \log X \leq_{disp} \log Y. \tag{5}$$

Thus, the star order and the dispersive order are in general not nested. However, under some conditions they are.

Lemma 7. *For nonnegative random variables, X and Y with distribution functions F and G, respectively,*

(i) if $X \leq_{FOSD} Y$, then $X \leq_* Y$ implies $X \leq_{disp} Y$.

[42] For a formal definition of these concepts, see Shaked and Shanthikumar (2007) or Ganuza and Penalva (2010).

[43] For further insights on the relation to other informativeness criteria, like *sufficiency* (Blackwell, 1951) or *accuracy*, respectively *effectiveness* (Lehmann, 1988; Persico, 2000) we refer the reader to the discussion in Ganuza and Penalva (2010).

(ii) if F and G are absolutely continuous with $F(0) = G(0) = 0$ and $f(0) \geq g(0) > 0$, then $X \leq_* Y$ implies $X \leq_{disp} Y$.

Figure 4 summarizes the relation of the three precision criteria and the sets of signals that are ordered in terms of any of these criteria.

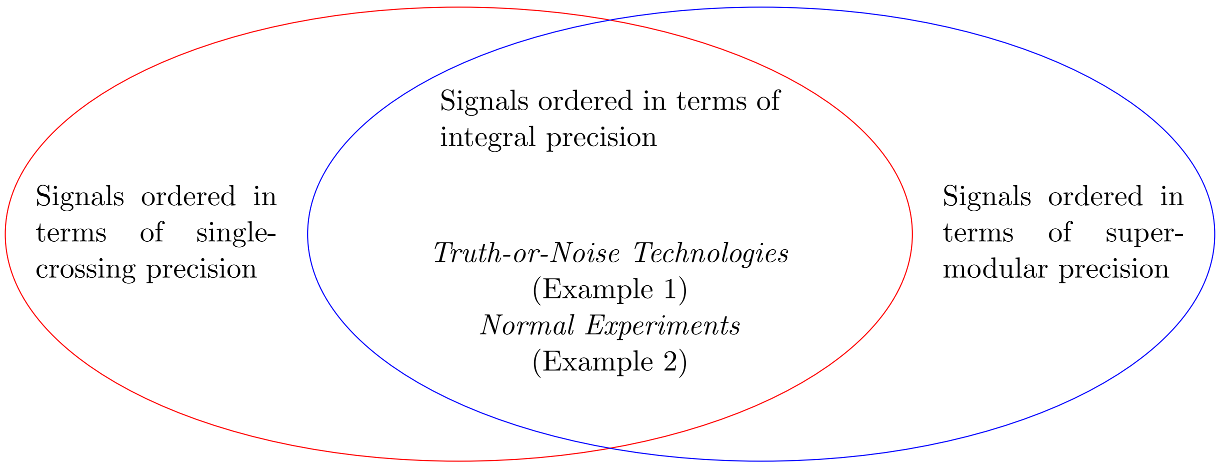


Figure 4: Illustration of the relation between the concepts of single-crossing precision, supermodular precision and integral precision.

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